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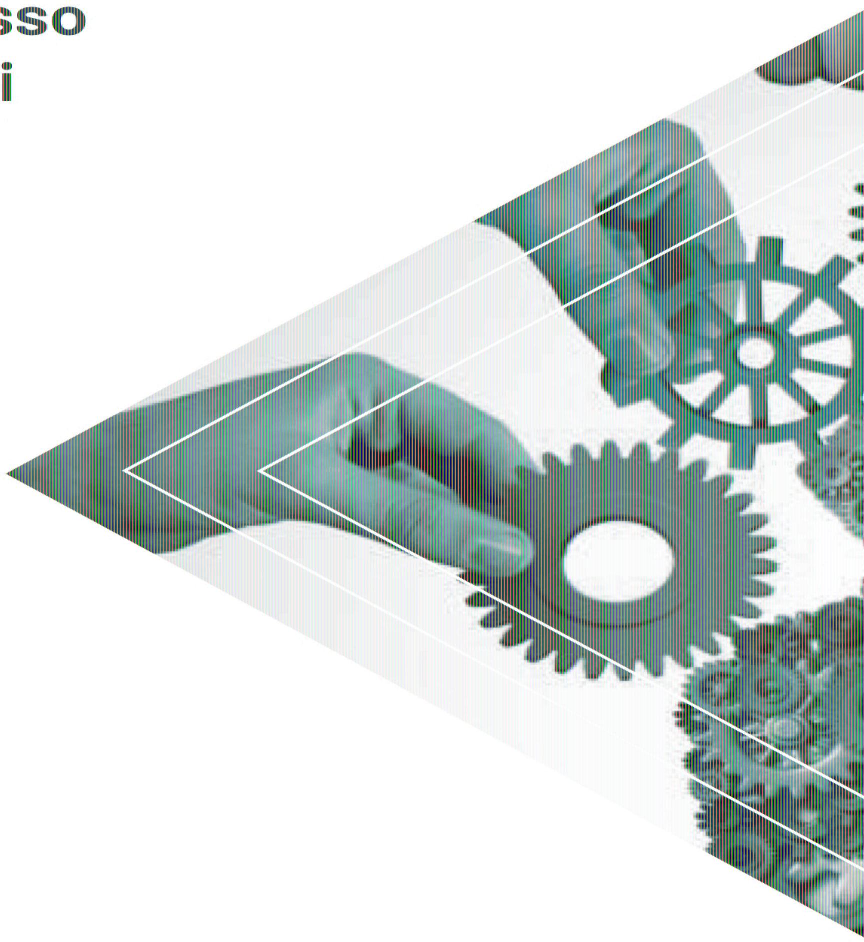

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Tax Rebate and Price Discrimination

Salvatore Capasso
Department of Business and Economics, University of Naples Parthenope, Italy
Department of Human and Social Studies, CNR, ITALY
salvatore.capasso@uniparthenope.it

Salvatore Ciucci
Department of Economics, University of Campania "Luigi Vanvitelli", ITALY
salvatore.ciucci@unicampania.it

ABSTRACT

Questo articolo studia l'effetto che le deduzioni fiscali possono avere sulla scelta di evadere le tasse. Nel modello teorico proposto l'evasione fiscale si concretizza attraverso la mancata emissione della fattura o ricevuta fiscale da parte del venditore, il quale opera in un mercato monopolistico. La scelta di rilasciare la ricevuta fiscale è condizionata dalla contrattazione che avviene tra compratore e venditore, che propone uno sconto sul prezzo da pagare, al fine di evitare l'emissione della ricevuta fiscale. In base alla propria moralità, i compratori decideranno se accettare o meno la proposta del venditore, nel caso dovessero rifiutare, la ricevuta fiscale costituirà la prova necessaria per usufruire di una deduzione fiscale. I risultati derivanti dell'analisi teorica proposta sono molteplici, viene dimostrato che ove possibile, una strategia di discriminazione di prezzo di terzo grado sarà la strategia ottimale adottata dal venditore; tale strategia si rivelerà più efficace nella lotta all'evasione fiscale, rispetto al meccanismo delle deduzioni fiscali. Emerge, inoltre, che l'entità delle deduzioni fiscali dipende dal grado di moralità dei compratori. Infine, è possibile derivare le condizioni, sotto le quali le deduzioni fiscali causano addirittura una riduzione del benessere sociale.

We study the effect of a tax rebate on tax evasion, in a third – degree price discrimination framework. In such a context, the monopolist contracts with the buyers a price discount, in exchange for not issuing the transaction receipt, hiding sales revenues from tax authority. Buyers in the economy are heterogeneous only in the honesty or tax morale, and keeping the purchase receipt, they are eligible for a tax rebate. We show that a price discrimination strategy will always lead to a reduction in tax evasion. The monopolist will always choose to discriminate, the aggregate quantity produced in the economy will be unchanged, and no effect on consumption will be generated. Furthermore, we prove that the tax rebate policy is strongly affected by the distribution of tax morale among population. The tax rebate could be a sub optimal policy and could lead to a decrease in social welfare.

Keywords: Tax rebate, Price discrimination, Tax evasion, Tax morale.
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1. Introduction

If we asked people why they pay taxes, the overwhelming majority would answer: because the government forces us to. The government can move people towards a fuller tax compliance, through two different procedures, a deterrence approach, based on audits, fines and criminal punishment, and a rewarding approach, based on a positive reward for individual taxpayers keeping purchase receipts. The literature has already widely shown that the reward treatments is an effective tool to fight against tax evasion and also a useful antirecession instrument, stimulating both consumption and social welfare; even many experimental studies suggest that the rewarding approach should be the most preferable strategy to curb tax evasion (Alm, McClelland and Schulze 1992; Berhan and Jenkins 2005).

Many countries in the world adopt a tax rebate policy. Just to name a few, European Union Member States have introduced tax reductions for expenses related to household repair, maintenance and improvement sector; China and Puerto Rico offer a monetary subsidy to buyers providing evidence of legal transaction, the state of Sao Paulo in Brazil even offers a tax rebate of up to thirty percent of the taxes collected in retail purchases, but also United States, United Kingdom, Ireland, India, Canada and New Zealand use a tax refund plan.

There are very few papers that analyze the effects of a tax rebate policy on individuals' tax compliance choice, considering tax evasion as the result of collusive behaviour between buyers and sellers. Piolatto (2015) calculates the optimal tax rebate policy that leads both shrinking tax evasion and growing tax proceeds; Immordino and Russo (2018) also get the same results by proposing a model of pure cooperative tax evasion, furthermore they show that the optimal rebate rate is increasing in tax evasion, and tax rebate policy becomes more expensive as tax evasion level falls. The novelty of this paper is the possibility of the seller to apply a price discrimination strategy, charging different price on the basis of the individual tax morale type. To the best of our knowledge, this is the first attempt to analyze the effects of a tax rebate policy, considering the impact of price discrimination in a cooperative tax evasion model.

Despite both theoretical and empirical literature tests tax rebate as one of the most important fiscal tools at the government's disposal, which can be used to restrain tax evasion, getting additional benefits, in terms of consumption and social welfare; it is evident that many countries do not adopt any tax rebate policy, preferring a deterrent approach, or if they do, they choose to adopt different policies depending on the related sector of the economy. Our analysis tries to provide a valid explanation of the functioning underlying the use of a tax rebate policy.

In our theoretical setting, tax evasion occurs when the buyer, not asking for the tax receipt, proof of the legal transaction, negotiates a price discount with the seller, which can charge different prices based on the population 's tax morale type. We derive a series of surprising results, when the seller has the possibility to apply a price discrimination strategy, which turns out to be the best pricing strategy, there is always a reduction in tax evasion. The optimal tax rebate policy is strongly affected by the distribution of the tax morale among the population, and under certain conditions it is optimal for the government to set a rebate rate equal to zero. Furthermore, no effects on consumption arise, and the social welfare is strongly affected by the dimension of population and tax morale, showing that in some cases the tax rebate policy is just an excellent tool to reward honest individuals and punish dishonest ones.

The paper now proceeds as follows. In Section 2 and in its subsections, we set up the basic model, characterize the best pricing strategy and the optimal tax rebate policy, and consider social welfare implications. Conclusions follow in Section 3.

2. The Model

The Economy consists of a continuum of identical buyers (consumers) normalized to unity, denoted by i, seller (monopolistic firm) and a government. All buyers in the economy, differ from each other, only by one dimension, the honesty or tax morale[1] θ_i, and $i \in [x; y; z]$, where x is the percentage of population endowed with a tax morale equal to θ_x, y is the percentage of population with tax morale equal to θ_y, and finally z is the percentage of buyers of type θ_z. There is only one firm in the market, operating as a monopolist, selling a homogeneous good or service at price p. Transactions with buyers can take place both legally, if the purchase or tax receipt is issued, and illegally otherwise. Sales are subject to an ad valorem tax T, and since by hypothesis, government cannot perform any investigation[2], seller can engage in tax evasion, underreporting the fraction of sales without tax receipt. Asking for the tax receipt is costly, buyers have to pay a full price $p(1+T)$ for it, because a negotiation occurs between buyer and seller, who is willing to apply a price discount to those buyers who do not request a purchase receipt. We suppose a perfect elastic supply[3], the sales tax is entirely borne by the buyers, basically, it is like the seller were supplying two bundles, one that includes the good or service and the tax receipt at price $p(1+T)$, and another one, which includes only the good or service, and therefore also the collaboration, which allows the seller to shelter part of sales profits from taxation, at price p. The government collecting taxes from the sales certified by tax receipts, cannot randomly audit the seller, and to encourage buyers to request purchase receipts, it offers a monetary subsidy or a tax rebate $\tau \in [0, T[$ for those who keep the trace of the transaction. In addition to the fact that there are no tax investigations, we also assume that T is exogenous and fixed, so that the only way to influence buyers' behavior is through the tax rebate[4].

The buyers, who choose to collaborate in the seller's tax evasion, by not asking for the tax receipt, and which from now on, we will call bad buyers, derive their utility from the following function:

$$u_{b,i}(q_{b,i}; p) = (v - \theta_i p)q_{b,i} - \frac{{q_{b,i}}^2}{2} \tag{1}$$

Instead, buyers who request purchase receipts and can benefit of the monetary subsidy or tax rebate, and which from now on, we will call good buyers, derive their utility, from the following function:

$$u_{g,i}(q_{g,i}; p) = \big(v - p(1+T-\tau)\big)q_{g,i} - \frac{{q_{g,i}}^2}{2} \tag{2}$$

v is the individual willingness to pay, which is equal for all buyers, $q_{b,i}$ and $q_{g,i}$ are the quantity demanded by bad and good buyers of type i respectively, θ_i represents what the literature defines the "psychic cost of tax evasion"[5].

Differentiating Eq. (1) and (2) with respect to the demanded quantity, and solving for it:

$$q^*_{b,i} = v - \theta_i p \tag{3}$$
$$q^*_{g,i} = v - p(1+T-\tau) \tag{4}$$

$q^*_{b,i}$ and $q^*_{g,i}$ are the optimal quantities demanded by bad and good buyers of type i. Note that, both the optimal quantity and the utility of good buyers, are not affected by the individual type; because the tax moral affects only bad buyers, therefore good buyers of each type choose the same quantity and get the same level of utility. For this reason, from now on, to indicate the demanded quantities and the utility of good buyers, we will only write q_g and u_g.

Substituting Eq. (3) and (4) in the utility functions:

[1] See Gordon 1989; Myles and Naylor 1996; Andreoni, Erard and Feinstein 1998; Feld and Frey 2002; Orviska and Hudson 2003; Traxler 2010; Hug and Sporri 2011.
[2] The hypothesis that there are no tax investigations does not influence the results of our analysis in the slightest, it only helps to simplify the notation and calculations.
[3] This hypothesis, even if removed, does not influence in any way the results achieved by the model.
[4] Tax policy is often blocked, so the tax rebate is used precisely to modify the net tax rate, or to modify the tax rate of a specific category of expenditure.
[5] Gordon (1989) was the first to introduce the concept of psychic costs, understood as anxiety and self-image as well as social stigma that may be associated with tax evasion.

$$u_{b,i}(q_{b,i}^*; p) = \frac{1}{2}(v - \theta_i p)^2 \quad (5)$$

$$u_g(q_g^*; p) = \frac{1}{2}\big(v - p(1 + T - \tau)\big)^2 \quad (6)$$

Given their morality, each agent chooses to be a bad buyer, if the following condition is satisfied:

$$u_{b,i}(q_{b,i}^*; p) > u_g(q_g^*; p) \quad (7)$$

Otherwise, agents choose to be good buyers.
The Eq. (7) can be rewritten as:

$$1 \leq \theta_i < 1 + T - \tau \quad (8)$$

The previous equation represents the existence condition of tax evasion.
Now, assume that:

a. $\theta_x = 1$
b. $1 < \theta_y < 1 + T$
c. $\theta_z \geq 1 + T$

It is easy to notice that, condition sub a) always satisfies the inequality (8), while condition sub c) never satisfies the above inequality. This means that, for each value of the rebate rate $\tau \in [0, T[$; the percentage x of the population will always choose to be bad buyers, as opposed to the z percent of the population that will always ask for the tax receipt. Instead, individuals of type θ_y, will request purchase receipt only in exchange for an appropriate amount of tax discount[6], therefore, they will choose to be good buyers only if:

$$1 + T - \theta_y \leq \tau < T \quad (9)$$

Our assumptions about the individuals' tax morale, fit well to reality, because, as empirical works show, full tax compliance as well as full tax evasion condition is never satisfied.
In the next subsections, we analyze the optimal choice of the government and firm.

2.1 No price discrimination

Supposing that to reduce tax evasion, the government sets a rebate rate τ in the interval $[1 + T - \theta_y; T[$ so that only θ_x type agents choose to be bad buyers. If the firm does not adopt any price discrimination strategy, we use the superscript MTR (monopolist tax rebate) to identify such a scheme, it has to set a single monopolistic price p^{MTR}, which is charged to each type of agent, and for simplicity, we assume zero marginal costs. The aggregate demand Q^{MTR} is therefore:

$$Q^{MTR} = x\big(q_{b,x}^{MTR^*}\big) + (y + z)\big(q_g^{MTR^*}\big) \quad (10)$$

Using Eq. (3) and (4), we can write the firm's maximization problem as:

$$\begin{aligned} \max_{p^{MTR}} \Pi^{MTR} &= p^{MTR} Q^{MTR} \\ &= [x(v - \theta_x p^{MTR}) + (y + z)(v - (1 + T - \tau)p^{MTR})]\, p^{MTR} \end{aligned} \quad (11)$$

Solving and rearranging, we get:

$$p^{MTR^*} = \frac{v}{2[x + (y + z)(1 + T - \tau)]} = \frac{v}{2[1 + (1 - x)(T - \tau)]} \quad (12)$$

[6] Note that in absence of the tax rebate program ($\tau = 0$), the percentage of bad buyers is $x + y$.

$$Q^{MTR}\left(p^{MTR^*}\right) = \frac{v}{2} \tag{13}$$

The government maximizes total tax proceeds TP^{MTR}, which are given by the sum of the firm's income taxes:

$$\max_{\tau} TP^{MTR} = (y+z)p^{MTR^*}q_g^{MTR^*}(T-\tau) \tag{14}$$
$$s.t.$$
$$1+T-\theta_y \leq \tau < T$$

The solutions of the previous problem are (See Appendix A):

1. $\tau_1^* = 1+T-\theta_y$ if $x \leq \frac{1}{3}+\frac{1}{3(\theta_y-1)}$
2. $\tau_2^* = T-\frac{1}{3x-1}$ if $x \geq \frac{1}{3}+\frac{1}{3(\theta_y-1)}$

Since the firm's profit depends on the optimal tax rebate rate, and using Eq. (12) and (13), the monopolistic profit function assumes the following forms (See Appendix B):

$$\Pi^{MTR}(\tau_1^*) = \frac{v^2}{4}\cdot\frac{1}{\left(1+(1-x)(\theta_y-1)\right)} \tag{15}$$

$$\Pi^{MTR}(\tau_2^*) = \frac{v^2}{4}\cdot\frac{3x-1}{2x} \tag{16}$$

2.2 Price discrimination

Now, consider that the monopolist implements a price discrimination strategy, we use the superscript DM (discriminating monopolist) to identify such a regime, based on the agents' type, supplying four consumption bundles[7]: $\left(p_{b,x}; q_{b,x}^{DM}\right)$, $\left(p_{b,y}; q_{b,y}^{DM}\right)$, $\left(p_{b,z}; q_{b,z}^{DM}\right)$, $\left(p_g; q_g^{DM}\right)$.

We are referring to third-degree discrimination, where the monopolist is aware of the total (or average) demand of each single segment into which the market has been divided, but does not know the demand of each individual. Therefore, the tax morale $(\theta_x, \theta_y, \theta_z)$ of the population is a public information, while agent's type is a private information.

Discriminating, the monopolist charges agents' different prices, it is like dealing with individual demand functions, so the discriminating monopolist faces the following maximization problems:

$$\max_{p_g} \Pi_g^{DM} = p_g\, q_g^{DM^*} = vp_g - (1+T-\tau)p_g^2 \tag{17}$$

$$\max_{p_{b,i}} \Pi_{b,i}^{DM} = p_{b,i}\, q_{b,i}^{DM^*} = vp_{b,i} - \theta_i p_{b,i}^2 \tag{18}$$

Solving problems (17) and (18), and using Eq. (3) and (4), we have that:

$$p_g^* = \frac{v}{2(1+T-\tau)} \tag{19}$$

$$p_{b,i}^* = \frac{v}{2\theta_i} \tag{20}$$

$$q_g^{DM^*} = q_{b,i}^{DM^*} = \frac{v}{2} \tag{21}$$

[7] Note that, good buyers of each type choose the same consumption bundle, since the utility of good buyers is not affected by agent's type.

It is easy to see that $q_g^{DM^*} = q_{b,x}^{DM^*} = q_{b,y}^{DM^*} = q_{b,z}^{DM^*}$; the quantity sold in any case is the same, as well as the agents' disutility $\theta_x p_{b,x}^* = \theta_y p_{b,y}^* = \theta_z p_{b,z}^* = p_g^*(1 + T - \tau) = \frac{v}{2}$. This implies that:

$$u_g^{DM}\left(q_g^{DM^*}; p_g^*\right) = u_{b,x}^{DM}\left(q_{b,x}^{DM^*}; p_{b,x}^*\right) = u_{b,y}^{DM}\left(q_{b,y}^{DM^*}; p_{b,y}^*\right) = u_{b,z}^{DM}\left(q_{b,z}^{DM^*}; p_{b,z}^*\right) = \frac{v^2}{8} \quad (22)$$

Using conditions (19), (20), (21) and hypotheses a), b), c), we can highlight the following results:

- $p_{b,x}^* > p_g^* \Rightarrow \frac{v}{2} > \frac{v}{2(1+T-\tau)} \Rightarrow T > \tau$
- $p_{b,z}^* < p_g^* \Rightarrow \frac{v}{2\theta_z} < \frac{v}{2(1+T-\tau)} \Rightarrow \theta_z > 1 + T - \tau$
- $p_{b,y}^* > p_g^* \Rightarrow \frac{v}{2\theta_y} > \frac{v}{2(1+T-\tau)} \; if \; \tau < 1 + T - \theta_y$
- $p_{b,y}^* \leq p_g^* \Rightarrow \frac{v}{2\theta_y} \leq \frac{v}{2(1+T-\tau)} \; if \; \tau \geq 1 + T - \theta_y$

Moreover, since by hypothesis $\theta_z > \theta_y > \theta_x = 1$, then $p_{b,x}^* > p_{b,y}^* > p_{b,z}^*$.

Agents of type θ_z, are indifferent between buying with or without the tax receipt, but since $p_{b,z}^* < p_g^*$, for the monopolist it is not profitable to set $p_{b,z}^*$(considering also that the quantities sold in each case are equal, $q_g^{DM^*} = q_{b,z}^{DM^*}$). For the same reasons, if $p_{b,y}^* \leq p_g^*$, the monopolist does not set even $p_{b,y}^*$. The monopolist always sets $p_{b,x}^*$, that is always greater than p_g^* (since by hypothesis the amount of the rebate rate has to be lower than the tax rate); θ_x type agents are indifferent between asking for the tax receipt or not; therefore, the monopolist charges a price $p_{b,x}^* - \varepsilon$; with ε infinitely small, to encourage θ_x type agents to be bad buyers.

Instead, if $p_{b,y}^* > p_g^*$ (given that θ_y type agents are indifferent between buying without tax receipt at price $p_{b,y}^*$ or buying with tax receipt at price p_g^*), the monopolist can set a price $p_{b,y}^* - \varepsilon$; with ε infinitely small, to encourage θ_y type agents to be bad buyers, getting greater earnings, but since $p_{b,x}^* > p_{b,y}^*$, θ_x type agents would mimic θ_y type agents. In this case, θ_x and θ_y type agents choose to be bad buyers, buying at the price $p_{b,y}^*$, while θ_z type agents choose to be good buyers buying at price p_g^*, hence the monopolist's profit is $\Pi^{DM}\left(p_{b,x}^*; p_{b,y}^*; p_g^*\right) = (x + y)p_{b,y}^* q_{b,y}^{DM^*} + z p_g^* q_g^{DM^*}$.

If the monopolist, to avoid adverse selection problems, sets only two prices, $p_{b,x}^*$ and p_g^*; so that only θ_x type agents choose to be bad buyers, his profit becomes $\Pi^{DM}\left(p_{b,x}^*; p_g^*\right) = x p_{b,x}^* q_{b,x}^{DM^*} + (y + z) p_g^* q_g^{DM^*}$.

Since $\Pi^{DM}\left(p_{b,x}^*; p_g^*\right) > \Pi^{DM}\left(p_{b,x}^*; p_{b,y}^*; p_g^*\right)$ (See Appendix C for the proof), even if $p_{b,y}^* > p_g^*$, it is more convenient for the monopolist to set only prices $p_{b,x}^*$ and p_g^*; θ_y and θ_z type agents always asking for the tax receipt, while θ_x type agents choose to be bad buyers, regardless of the rebate rate τ. Therefore, the profit of the discriminating monopolist is:

$$\Pi^{DM} = \frac{v^2}{4}\left[\frac{1 + x(T - \tau)}{1 + T - \tau}\right] \quad (23)$$

The government maximization problem assumes the following form:

$$\max_{\tau} TP^{DM} = (y + z) p_g^* q_g^{DM^*}(T - \tau) \quad (23)$$

Solving that, and doing some substitutions (See Appendix D), we get:

$$\tau_3^* = 0 \quad (24)$$

Eq. (24) means that a price discrimination strategy leads to a reduction in tax evasion, even if the government does not adopt any tax rebate program.

2.3 No tax rebate and no price discrimination

In the event that government does not endorse any tax rebate program and the firm does not discriminate, we use the superscript NTR (no tax rebate) to identify such a scheme, both θ_x and θ_y type agents choose to be bad buyers, then the aggregate demand Q^{NTR} is:

$$Q^{NTR} = x\left(q_{b,x}^{NTR^*}\right) + y\left(q_{b,y}^{NTR^*}\right) + z\left(q_g^{NTR^*}\right) \tag{26}$$

Using Eq. (3) and (4), the firm's maximization problem becomes:

$$\begin{aligned} \max_{p^{NTR}} \Pi^{NTR} &= p^{NTR} Q^{NTR} \\ &= \left[x(v - \theta_x p^{NTR}) + y\left(v - \theta_y p^{NTR}\right) \right. \\ &\quad \left. + z(v - (1+T)p^{NTR})\right] p^{NTR} \end{aligned} \tag{27}$$

Solving and rearranging:

$$p^{NTR^*} = \frac{v}{2\left[x + y\theta_y + z(1+T)\right]} = \frac{v}{2\left[1 + T(1-x) - y\left(1 + T - \theta_y\right)\right]} \tag{28}$$

$$Q^{NTR}\left(p^{NTR^*}\right) = \frac{v}{2} \tag{29}$$

$$\Pi^{NTR} = \frac{v^2}{4\left[1 + T(1-x) - y\left(1 + T - \theta_y\right)\right]} \tag{30}$$

Using instead Eq. (5), (6) and (28), we can write the utility of each type of agent:

$$u_{b,x}^{NTR}\left(q_{b,x}^{NTR^*}; p^{NTR^*}\right) = \frac{v^2}{8} \cdot \frac{\left[1 + 2\left[T(1-x) - y\left(1 + T - \theta_y\right)\right]\right]^2}{\left[1 + T(1-x) - y\left(1 + T - \theta_y\right)\right]^2} \tag{31}$$

$$u_{b,y}^{NTR}\left(q_{b,y}^{NTR^*}; p^{NTR^*}\right) = \frac{v^2}{8} \cdot \frac{\left[2\left[1 + T(1-x) - y\left(1 + T - \theta_y\right)\right] - \theta_y\right]^2}{\left[1 + T(1-x) - y\left(1 + T - \theta_y\right)\right]^2} \tag{32}$$

$$u_g^{NTR}\left(q_g^{NTR^*}; p^{NTR^*}\right) = \frac{v^2}{8} \cdot \frac{\left[1 + T(1-2x) - 2y\left(1 + T - \theta_y\right)\right]^2}{\left[1 + T(1-x) - y\left(1 + T - \theta_y\right)\right]^2} \tag{33}$$

2.4 Optimal pricing strategy and tax rebate policy

Now, let's see under what conditions the monopolist chooses the best pricing strategy. If the government does not adopt any tax rebate program, the monopolist chooses to discriminate only if $\Pi^{DM}(\tau_3^*) > \Pi^{NTR}$, tax evasion is reduced, and it is not profitable for the government to set any positive value of the rebate rate, since in that case, the monopolist would continue to discriminate $\left(\frac{\partial \Pi^{DM}}{\partial \tau} > 0\right)$[8], and by doing so the government would reduce tax revenues, since $\frac{\partial TP^{DM}}{\partial \tau} < 0$ (See Appendix D). The condition $\Pi^{DM}(\tau_3^*) > \Pi^{NTR}$, is satisfied when $y < \frac{xT^2(1-x)}{(1+T-\theta_y)(1+xT)}$ (See Appendix E); therefore, for such a dimension of y, there is a reduction in tax evasion, even without a tax rebate.

If instead $y > \frac{xT^2(1-x)}{(1+T-\theta_y)(1+xT)}$; in order to reduce tax evasion, government has to set a certain value of the rebate rate, τ_1^* or τ_2^* compatibly with the dimension of x; but in either case, the monopolist will choose to discriminate, since $\Pi^{DM}(\tau_1^*) > \Pi^{MTR}(\tau_1^*)$ and $\Pi^{DM}(\tau_2^*) > \Pi^{MTR}(\tau_2^*)$ (See Appendix

[8] $\frac{\partial \Pi^{DM}}{\partial \tau} = \frac{v^2}{4} \frac{-x[1+T-\tau]+1+x(T-\tau)}{[1+T-\tau]^2} = \frac{v^2}{4} \frac{1-x}{[1+T-\tau]^2} > 0$

E). Since $\frac{\partial TP^{DM}}{\partial \tau} < 0$, it is convenient for the government to set the minimum value of the rebate rate, that allows tax evasion reduction. This value is always τ_1^* (See Appendix A).

At this point, government could reduce tax evasion, in a cheaper way, by setting a value of the rebate rate $\tau_4^* = \frac{y(1+T-\theta_y)(1+xT)-x(1-x)T^2}{(1-x)(1-xT)+xy(1+T-\theta_y)}$, such that $\Pi^{DM}(\tau_4^*) \cong \Pi^{NTR}$(See Appendix E), extracting all the surplus, which otherwise would belong to the monopolist.

Obviously, the government will choose the lower value between τ_1^* and τ_4^*. If $y < \frac{(1-x)[1+T-\theta_y+xT(\theta_y-1)]}{(1+T-\theta_y)[1+x(\theta_y-1)]}$ then $\tau_4^* < \tau_1^*$ and vice versa (See Appendix E).

The table 1 summarizes the results just obtained:

Tab. 1 – The optimal rebate rate based on the size of individuals of type θ_y

Dimension of y	Optimal rebate rate
$y < \frac{xT^2(1-x)}{(1+T-\theta_y)(1+xT)}$	τ_3^*
$\frac{xT^2(1-x)}{(1+T-\theta_y)(1+xT)} < y < \frac{(1-x)[1+T-\theta_y+xT(\theta_y-1)]}{(1+T-\theta_y)[1+x(\theta_y-1)]}$	τ_4^*
$y > \frac{(1-x)[1+T-\theta_y+xT(\theta_y-1)]}{(1+T-\theta_y)[1+x(\theta_y-1)]}$	τ_1^*

Source: Authors' own elaboration

It is clear, that the distribution of individual's type among the population, strongly affects the tax rebate policy. The main result is that, for low values of y, tax evasion decreases even if the government does not approve any tax rebate program, indeed, it becomes even optimal not to offer any tax rebate, and this could provide an excellent explanation for the lack of government intervention, in terms of tax rebate policy. It should also be noted that in any case the monopolist chooses to discriminate, and the tax rebate does not affect consumption, since the aggregate quantity in each regime is practically unchanged $Q^{MTR}(p^{MTR^*}) = Q^{NTR}(p^{NTR^*}) = Q^{DM}(p_{b,x}^*; p_g^*) = \frac{v}{2}$.

2.5 Social welfare analysis

Government intervention to reduce tax evasion can induce a change in utility function of the agents. Considering a utilitarian social welfare function (SWF), given by the sum of the utility of each individual, we get that the net change in social welfare, due to the tax rebate, is strongly affected by the dimension of population and tax morale (See Appendix F for the proof):

$$\Delta SWF = x[u_{b,x}^{DM}(q_{b,x}^{DM^*}; p_{b,x}^*) - u_{b,x}^{NTR}(q_{b,x}^{NTR^*}; p^{NTR^*})] + y[u_g^{DM}(q_g^{DM^*}; p_g^*) - u_{b,y}^{NTR}(q_{b,y}^{NTR^*}; p^{NTR^*})] + z[u_g^{DM}(q_g^{DM^*}; p_g^*) - u_g^{NTR}(q_g^{NTR^*}; p^{NTR^*})] < 0 \quad (34)$$

If $0 < x < \frac{T-2y(1+T-\theta_y)+\sqrt{[T-2y(1+T-\theta_y)]^2+4y(1-y)(1+T-\theta_y)^2}}{2T}$.

On the contrary, if $\frac{T-2y(1+T-\theta_y)+\sqrt{[T-2y(1+T-\theta_y)]^2+4y(1-y)(1+T-\theta_y)^2}}{2T} < x < 1$; then $\Delta SWF > 0$. Therefore, social welfare could be even reduced, showing that sometimes, the use of the tax rebate to fighting against tax evasion, could be just a way to reward honest individuals and punish dishonest ones, since analysis shows that:

$$u_{b,x}^{DM}\left(q_{b,x}^{DM^*}; p_{b,x}^*\right) - u_{b,x}^{NTR}\left(q_{b,x}^{NTR^*}; p^{NTR^*}\right) < 0$$

$$u_g^{DM}\left(q_g^{DM^*}; p_g^*\right) - u_{b,y}^{NTR}\left(q_{b,y}^{NTR^*}; p^{NTR^*}\right) > 0 \text{ if } \theta_y > 1 + \frac{T(1-x-y)}{1-y}$$

$$u_g^{DM}\left(q_g^{DM^*}; p_g^*\right) - u_{b,y}^{NTR}\left(q_{b,y}^{NTR^*}; p^{NTR^*}\right) < 0 \text{ if } \theta_y < 1 + \frac{T(1-x-y)}{1-y}$$

$$u_g^{DM}\left(q_g^{DM^*}; p_g^*\right) - u_g^{NTR}\left(q_g^{NTR^*}; p^{NTR^*}\right) > 0$$

See Appendix F for the proof.

3. Conclusions

We have built a theoretical model to analyze the determinants of the tax rebate policy as an indirect mechanism to shrink tax evasion. In our model of cooperative tax evasion, the buyers decide whether to ask for the transaction receipt, thus getting a tax rebate, because by certifying their expenditure, they prevent the seller from hiding sales revenues. The seller is a monopolist, which can discriminate between buyers, on the basis of their tax morale type, by charging different prices and by bargaining a price discount with buyers who do not ask for tax receipt.

We have shown that the choice of the optimal pricing strategy for the monopolist, as well as the best tax rebate policy for the government, depends on the tax morale distribution among the population. The price discrimination, which is the best pricing strategy, always leads to a reduction in tax evasion. The tax rebate policy could be suboptimal.

The most surprising result of our analysis is that the tax rebate has no effect on aggregate consumption, could lead to a reduction in social welfare.

All the above findings could explain the rationale behind the different tax rebate policies adopted around the world.

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APPENDIX A

The maximization problem is:

$$\max_{\tau} TP^{MTR} = (y+z)p^{MTR^*}q_g^{MTR^*}(T-\tau)$$

$$s.t.$$

$$1+T-\theta_y \le \tau < T$$

$$p^{MTR^*} = \frac{v}{2[1+(1-x)(T-\tau)]}$$

$$q_g^{MTR^*} = \left(v-(1+T-\tau)p^{MTR^*}\right) = \left(v - \frac{v(1+T-\tau)}{2[1+(1-x)(T-\tau)]}\right)$$
$$= v\left(\frac{2[1+(1-x)(T-\tau)]-(1+T-\tau)}{2[1+(1-x)(T-\tau)]}\right)$$
$$= \frac{v}{2}\left(\frac{2+2(1-x)(T-\tau)-1-T+\tau}{1+(1-x)(T-\tau)}\right) = \frac{v}{2}\left(\frac{1+2(1-x)(T-\tau)-(T-\tau)}{1+(1-x)(T-\tau)}\right)$$
$$= \frac{v}{2}\left(\frac{1+(1-2x)(T-\tau)}{1+(1-x)(T-\tau)}\right)$$

We can rewrite the maximization problem as:

$$\max_{\tau} TP^{MTR} = \frac{v^2}{4}\left(\frac{1+(1-2x)(T-\tau)}{[1+(1-x)(T-\tau)]^2}\right)(1-x)(T-\tau)$$
$$s.t.$$
$$1+T-\theta_y \le \tau < T$$

The first order condition is:

$$\frac{\partial TP^{MTR}}{\partial \tau} = \frac{v^2}{4}\frac{f'(\tau)g(\tau)-f(\tau)g'(\tau)}{[g(\tau)]^2}$$

$$f(\tau) = [1+(1-2x)(T-\tau)](1-x)(T-\tau)$$

$$g(\tau) = [1+(1-x)(T-\tau)]^2$$

$$f'(\tau) = -(1-2x)(1-x)(T-\tau)-(1-x)[1+(1-2x)(T-\tau)]$$
$$= -(1-x)[1+2(1-2x)(T-\tau)]$$

$$g'(\tau) = -2(1-x)[1+(1-x)(T-\tau)]$$

$$\frac{\partial TP^{MTR}}{\partial \tau} = \frac{v^2}{4}\frac{\begin{array}{c}-(1-x)[1+2(1-2x)(T-\tau)][1+(1-x)(T-\tau)]^2+\\ +2(1-x)^2(T-\tau)[1+(1-2x)(T-\tau)][1+(1-x)(T-\tau)]\end{array}}{[1+(1-x)(T-\tau)]^4}$$

$$\frac{\partial TP^{MTR}}{\partial \tau}$$
$$= \frac{v^2}{4}\frac{(1-x)\{-[1+2(1-2x)(T-\tau)][1+(1-x)(T-\tau)]+2(1-x)(T-\tau)[1+(1-2x)(T-\tau)]\}}{[1+(1-x)(T-\tau)]^3}$$

$\frac{\partial TP^{MTR}}{\partial \tau} = 0$ when $\{-[1+2(1-2x)(T-\tau)][1+(1-x)(T-\tau)] + 2(1-x)(T-\tau)[1+(1-2x)(T-\tau)]\} = 0$

$$-1-(1-x)(T-\tau)-2(1-2x)(T-\tau)-2(1-x)(1-2x)(T-\tau)^2+2(1-x)(T-\tau) + 2(1-x)(1-2x)(T-\tau)^2 = 0$$

$$-1+(1-x)(T-\tau)-2(1-2x)(T-\tau)=0$$

$$-1-(T-\tau)[-1+x+2-4x]=0$$

$$-1-(T-\tau)[1-3x]=0$$

$$\tau^* = T + \frac{1}{1-3x}$$

$$\frac{\partial TP^{MTR}}{\partial \tau} > 0 \Rightarrow -1-(T-\tau)[1-3x] > 0$$

$$\frac{\partial TP^{MTR}}{\partial \tau} < 0 \Rightarrow -1-(T-\tau)[1-3x] < 0$$

- If $[1-3x] > 0 \Rightarrow x < \frac{1}{3}$

$$\tau^* > T$$

$$\frac{\partial TP^{MTR}}{\partial \tau} > 0 \Rightarrow \tau > T + \frac{1}{1-3x}$$

$$\frac{\partial TP^{MTR}}{\partial \tau} < 0 \Rightarrow \tau < T + \frac{1}{1-3x}$$

The function TP^{MTR} is increasing to the right of τ^* and it is decreasing to the left of τ^*; therefore τ^* is a point of minimum. Since $1+T-\theta_y \leq \tau^* < T$; the value that maximizes TP^{MTR} is $\tau_1^* = 1 + T - \theta_y$.

- If $[1-3x] < 0 \Rightarrow x > \frac{1}{3}$

$$\tau^* < T$$

$$\frac{\partial TP^{MTR}}{\partial \tau} > 0 \Rightarrow \tau < T + \frac{1}{1-3x}$$

$$\frac{\partial TP^{MTR}}{\partial \tau} < 0 \Rightarrow \tau > T + \frac{1}{1-3x}$$

The function TP^{MTR} is decreasing to the right of τ^* and it is increasing to the left of τ^*; therefore τ^* is a point of maximum. Since $1+T-\theta_y \leq \tau^* < T$; then $\tau_2^* = T - \frac{1}{3x-1} \geq 1 + T - \theta_y$.

$$T - \frac{1}{3x-1} \geq 1 + T - \theta_y \Rightarrow 3x - 1 \geq \frac{1}{\theta_y - 1} \Rightarrow x \geq \frac{1}{3} + \frac{1}{3(\theta_y - 1)}$$

Therefore, summarizing, the final solutions of the maximization problem are:

$\tau_1^* = 1 + T - \theta_y$ if $x \leq \frac{1}{3} + \frac{1}{3(\theta_y - 1)}$

$\tau_2^* = T - \frac{1}{3x-1}$ if $x \geq \frac{1}{3} + \frac{1}{3(\theta_y - 1)}$

APPENDIX B

We have to show that:

$$\Pi^{MTR}(\tau_1^*) = \frac{v^2}{4} \cdot \frac{1}{\left(1 + (1-x)\left(\theta_y - 1\right)\right)}$$

$$\Pi^{MTR}(\tau_2^*) = \frac{v^2}{4} \cdot \frac{3x-1}{2x}$$

Given that:

$$\Pi^{MTR} = p^{MTR} Q^{MTR}$$

$$p^{MTR^*} = \frac{v}{2[x + (y+z)(1 + T - \tau)]} = \frac{v}{2[1 + (1-x)(T - \tau)]}$$

$$Q^{MTR}\left(p^{MTR^*}\right) = \frac{v}{2}$$

$$\tau_1^* = 1 + T - \theta_y$$

$$\tau_2^* = T - \frac{1}{3x-1}$$

We can write the monopolist's profit as:

$$\Pi^{MTR} = \frac{v^2}{4} \cdot \frac{1}{[1 + (1-x)(T - \tau)]}$$

$$\Pi^{MTR}(\tau_1^*) = \frac{v^2}{4} \cdot \frac{1}{\left[1 + (1-x)\left(T - 1 - T + \theta_y\right)\right]} = \frac{v^2}{4} \cdot \frac{1}{\left(1 + (1-x)\left(\theta_y - 1\right)\right)}$$

$$\Pi^{MTR}(\tau_2^*) = \frac{1}{\left[1 + (1-x)\left(T - T + \frac{1}{3x-1}\right)\right]} = \frac{1}{\left[\frac{3x - 1 + 1 - x}{3x-1}\right]} = \frac{v^2}{4} \cdot \frac{3x-1}{2x}$$

APPENDIX C

We have to show that:

$$\Pi^{DM}\left(p_{b,x}^{*};p_{g}^{*}\right) > \Pi^{DM}\left(p_{b,x}^{*};p_{b,y}^{*};p_{g}^{*}\right)$$

And we know:

$$\Pi^{DM}\left(p_{b,x}^{*};p_{g}^{*}\right) = xp_{b,x}^{*}q_{b,x}^{DM^{*}} + (y+z)p_{g}^{*}q_{g}^{DM^{*}}$$

$$\Pi^{DM}\left(p_{b,x}^{*};p_{b,y}^{*};p_{g}^{*}\right) = (x+y)p_{b,y}^{*}q_{b,y}^{DM^{*}} + zp_{g}^{*}q_{g}^{DM^{*}}$$

$$p_{g}^{*} = \frac{v}{2(1+T-\tau)}$$

$$p_{b,i}^{*} = \frac{v}{2\theta_i}$$

$$q_{g}^{DM^{*}} = q_{b,i}^{DM^{*}} = \frac{v}{2}$$

Therefore:

$$\Pi^{DM}\left(p_{b,x}^{*};p_{g}^{*}\right) = x\cdot\frac{v^2}{4} + (y+z)\frac{v^2}{4(1+T-\tau)} = \frac{v^2}{4}\cdot\left[x+\frac{y+z}{(1+T-\tau)}\right]$$

$$\Pi^{DM}\left(p_{b,x}^{*};p_{b,y}^{*};p_{g}^{*}\right) = (x+y)\cdot\frac{v^2}{4\theta_y} + z\cdot\frac{v^2}{4(1+T-\tau)} = \frac{v^2}{4}\cdot\left[\frac{x+y}{\theta_y}+\frac{z}{(1+T-\tau)}\right]$$

$$\Pi^{DM}\left(p_{b,x}^{*};p_{g}^{*}\right) > \Pi^{DM}\left(p_{b,x}^{*};p_{b,y}^{*};p_{g}^{*}\right) \Rightarrow \frac{v^2}{4}\cdot\left[x+\frac{y+z}{(1+T-\tau)}\right] > \frac{v^2}{4}\cdot\left[\frac{x+y}{\theta_y}+\frac{z}{(1+T-\tau)}\right]$$

$$x+\frac{y}{(1+T-\tau)} > \frac{x+y}{\theta_y}$$

$$\theta_y(1+T-\tau)x + y\theta_y > (1+T-\tau)(x+y)$$

$$x\theta_y(1+T) - \tau\theta_y x + y\theta_y > x(1+T) + y(1+T) - \tau x - \tau y$$

$$x(1+T)\left(\theta_y-1\right) + y\left[\theta_y-(1+T)\right] > \tau\left[x\left(\theta_y-1\right)-y\right]$$

$$\tau < \frac{x(1+T)\left(\theta_y-1\right) + y\left[\theta_y-(1+T)\right]}{\left[x\left(\theta_y-1\right)-y\right]}$$

In order to verify the previous inequality, let's first prove that the following inequality is always satisfied:

$$T < \frac{x(1+T)\left(\theta_y-1\right) + y\left[\theta_y-(1+T)\right]}{\left[x\left(\theta_y-1\right)-y\right]}$$

$$xT\left(\theta_y-1\right) - yT < xT\left(\theta_y-1\right) + x\left(\theta_y-1\right) + y\left(\theta_y-1\right) - yT$$

$$(x+y)\left(\theta_y-1\right) > 0$$

The previous inequality is always satisfied, because $\theta_y > 1$ by hypothesis.

Since, by hypothesis $\tau < T$, if $T < \frac{x(1+T)(\theta_y-1)+y[\theta_y-(1+T)]}{[x(\theta_y-1)-y]}$ is always satisfied, also $\tau < \frac{x(1+T)(\theta_y-1)+y[\theta_y-(1+T)]}{[x(\theta_y-1)-y]}$ is satisfied, therefore $\Pi^{DM}\left(p_{b,x}^{*};p_{g}^{*}\right) > \Pi^{DM}\left(p_{b,x}^{*};p_{b,y}^{*};p_{g}^{*}\right)$.

APPENDIX D

The maximization problem is of the form:

$$\max_{\tau} TP^{DM} = (y+z)p_g^* q_g^{DM^*}(T-\tau)$$
$$s.t.$$
$$\tau \geq 0$$

$$p_g^* = \frac{v}{2(1+T-\tau)}$$

$$q_g^{DM^*} = \frac{v}{2}$$

$$\max_{\tau} TP^{DM} = \frac{v^2(y+z)(T-\tau)}{4(1+T-\tau)}$$

$$\frac{\partial TP^{DM}}{\partial \tau} = \frac{v^2(y+z)}{4} \cdot \frac{-(1+T-\tau)+(T-\tau)}{(1+T-\tau)^2} = -\frac{v^2(y+z)}{4(1+T-\tau)^2} < 0$$

Since TP^{DM} is decreasing in τ, $\tau^* = 0$.

APPENDIX E

We have to prove:

$$\Pi^{DM}(\tau_3^*) > \Pi^{NTR}$$

Given that:

$$\Pi^{DM} = \frac{v^2}{4}\left[\frac{1 + x(T - \tau)}{1 + T - \tau}\right]$$

$$\tau_3^* = 0$$

$$\Pi^{DM}(\tau_3^*) = \frac{v^2}{4}\left[\frac{1 + xT}{1 + T}\right]$$

$$\Pi^{NTR} = \frac{v^2}{4\left[1 + T(1 - x) - y\left(1 + T - \theta_y\right)\right]}$$

Therefore:

$$\frac{1 + xT}{1 + T} > \frac{1}{1 + T(1 - x) - y\left(1 + T - \theta_y\right)}$$

$$(1 + xT)\left[1 + T(1 - x) - y\left(1 + T - \theta_y\right)\right] > 1 + T$$

$$1 + xT + (1 + xT)(1 - x)T - y\left(1 + T - \theta_y\right)(1 + xT) > 1 + T$$

$$-T(1 - x) + (1 + xT)(1 - x)T > y\left(1 + T - \theta_y\right)(1 + xT)$$

$$T(1 - x)(1 + xT - 1) > y\left(1 + T - \theta_y\right)(1 + xT)$$

$$y < \frac{xT^2(1 - x)}{\left(1 + T - \theta_y\right)(1 + xT)}$$

- Now, let's prove:

$$\Pi^{DM}(\tau_1^*) > \Pi^{MTR}(\tau_1^*)$$

Given that:

$$\Pi^{DM} = \frac{v^2}{4}\left[\frac{1 + x(T - \tau)}{1 + T - \tau}\right]$$

$$\tau_1^* = 1 + T - \theta_y$$

$$\Pi^{DM}(\tau_1^*) = \frac{v^2}{4}\left[\frac{1 + x\left(\theta_y - 1\right)}{\theta_y}\right]$$

$$\Pi^{MTR}(\tau_1^*) = \frac{v^2}{4}\left[\frac{1}{\left(1 + (1 - x)\left(\theta_y - 1\right)\right)}\right]$$

$$\frac{v^2}{4}\left[\frac{1 + x\left(\theta_y - 1\right)}{\theta_y}\right] > \frac{v^2}{4}\left[\frac{1}{\left(1 + (1 - x)\left(\theta_y - 1\right)\right)}\right]$$

$$\left(1 + x\left(\theta_y - 1\right)\right)\left(1 + (1 - x)\left(\theta_y - 1\right)\right) > \theta_y$$

$$1 + (1 - x)\left(\theta_y - 1\right) + x\left(\theta_y - 1\right) + x(1 - x)\left(\theta_y - 1\right)^2 > \theta_y$$

$$1 + \theta_y - 1 - x\left(\theta_y - 1\right) + x\left(\theta_y - 1\right) + x(1 - x)\left(\theta_y - 1\right)^2 > \theta_y$$

$$x(1-x)\left(\theta_y-1\right)^2>0$$

Since, by hypothesis $(1-x)$ is always greater than zero, the previous inequality is satisfied.

- Let's prove:

$$\Pi^{DM}(\tau_2^*)>\Pi^{MTR}(\tau_2^*)$$

Given that:

$$\Pi^{DM}=\frac{v^2}{4}\left[\frac{1+x(T-\tau)}{1+T-\tau}\right]$$

$$\tau_2^*=T-\frac{1}{3x-1}$$

$$\Pi^{DM}(\tau_2^*)=\frac{v^2}{4}\left[\frac{1+x\left(\frac{1}{3x-1}\right)}{1+\frac{1}{3x-1}}\right]=\frac{v^2}{4}\left[\frac{\frac{3x-1+x}{3x-1}}{\frac{3x-1+1}{3x-1}}\right]=\frac{v^2}{4}\left[\frac{4x-1}{3x}\right]$$

$$\Pi^{MTR}(\tau_2^*)=\frac{v^2}{4}\left[\frac{3x-1}{2x}\right]$$

$$\frac{v^2}{4}\left[\frac{4x-1}{3x}\right]>\frac{v^2}{4}\left[\frac{3x-1}{2x}\right]$$

$$2x(4x-1)>3x(3x-1)$$

$$8x-2-9x+3>0$$

$$1-x>0$$

By hypothesis, $1-x$ is always greater than zero, therefore the previous inequality is satisfied.

- Now, we prove:

$$\Pi^{DM}(\tau_4^*)=\Pi^{NTR}$$

Given that:

$$\Pi^{DM}=\frac{v^2}{4}\left[\frac{1+x(T-\tau)}{1+T-\tau}\right]$$

$$\Pi^{NTR}=\frac{v^2}{4\left[1+T(1-x)-y\left(1+T-\theta_y\right)\right]}$$

We derive the value of τ_4^* such that $\Pi^{DM}=\Pi^{NTR}$.

$$\frac{v^2}{4}\left[\frac{1+x(T-\tau)}{1+T-\tau}\right]=\frac{v^2}{4\left[1+T(1-x)-y\left(1+T-\theta_y\right)\right]}$$

$$[1+x(T-\tau)]\left[1+T(1-x)-y\left(1+T-\theta_y\right)\right]=1+T-\tau$$

$$1+T(1-x)-y\left(1+T-\theta_y\right)+xT-x\tau+xT^2(1-x)-xT\tau(1-x)-xyT\left(1+T-\theta_y\right)+xy\tau\left(1+T-\theta_y\right)=1+T-\tau$$

$$\tau-x\tau-xT\tau(1-x)+xy\tau\left(1+T-\theta_y\right)=y\left(1+T-\theta_y\right)-x(1-x)T^2+xyT\left(1+T-\theta_y\right)$$

$$\tau\left[1-x-xT(1-x)+xy\left(1+T-\theta_y\right)\right]=y\left(1+T-\theta_y\right)(1+xT)-x(1-x)T^2$$

$$\tau_4^* = \frac{y(1+T-\theta_y)(1+xT) - x(1-x)T^2}{(1-x)(1-xT) + xy(1+T-\theta_y)}$$

- Now, we have to show that $\tau_4^* < \tau_1^*$ when $y < \frac{(1-x)[1+T-\theta_y+xT(\theta_y-1)]}{(1+T-\theta_y)[1+x(\theta_y-1)]}$

Given that:

$$\tau_4^* = \frac{y(1+T-\theta_y)(1+xT) - x(1-x)T^2}{(1-x)(1-xT) + xy(1+T-\theta_y)}$$

$$\tau_1^* = 1+T-\theta_y$$

Hence:

$$\frac{y(1+T-\theta_y)(1+xT) - x(1-x)T^2}{(1-x)(1-xT) + xy(1+T-\theta_y)} < 1+T-\theta_y$$

$$y(1+T-\theta_y)(1+xT) - x(1-x)T^2 < (1+T-\theta_y)[(1-x)(1-xT) + xy(1+T-\theta_y)]$$

$$y(1+T-\theta_y)(1+xT) - x(1-x)T^2 < (1+T-\theta_y)(1-x)(1-xT) + xy(1+T-\theta_y)^2$$

$$y(1+T-\theta_y)[1+xT-x(1+T-\theta_y)] < (1-x)[(1-xT)(1+T-\theta_y) + xT^2]$$

$$y(1+T-\theta_y)[1+x(\theta_y-1)] < (1-x)[1-xT+T-\theta_y+\theta_y xT]$$

$$y < \frac{(1-x)[1+T-\theta_y+xT(\theta_y-1)]}{(1+T-\theta_y)[1+x(\theta_y-1)]}$$

APPENDIX F

Let's prove that, if $0 < x < \frac{T-2y(1+T-\theta_y)+\sqrt{\left[T-2y(1+T-\theta_y)\right]^2+4y(1-y)(1+T-\theta_y)^2}}{2T}$ then:

$$\Delta SWF = x\left[u_{b,x}^{DM}\left(q_{b,x}^{DM^*};p_{b,x}^*\right)-u_{b,x}^{NTR}\left(q_{b,x}^{NTR^*};p^{NTR^*}\right)\right]+y\left[u_g^{DM}\left(q_g^{DM^*};p_g^*\right)-u_{b,y}^{NTR}\left(q_{b,y}^{NTR^*};p^{NTR^*}\right)\right] + z\left[u_g^{DM}\left(q_g^{DM^*};p_g^*\right)-u_g^{NTR}\left(q_g^{NTR^*};p^{NTR^*}\right)\right] < 0$$

Given that:

$$u_g^{DM}\left(q_g^{DM^*};p_g^*\right)=u_{b,x}^{DM}\left(q_{b,x}^{DM^*};p_{b,x}^*\right)=u_{b,y}^{DM}\left(q_{b,y}^{DM^*};p_{b,y}^*\right)=u_{b,z}^{DM}\left(q_{b,z}^{DM^*};p_{b,z}^*\right)=\frac{v^2}{8}$$

$$u_{b,x}^{NTR}\left(q_{b,x}^{NTR^*};p^{NTR^*}\right)=\frac{v^2}{8}\cdot\frac{\left[1+2\left[T(1-x)-y\left(1+T-\theta_y\right)\right]\right]^2}{\left[1+T(1-x)-y\left(1+T-\theta_y\right)\right]^2}$$

$$u_{b,y}^{NTR}\left(q_{b,y}^{NTR^*};p^{NTR^*}\right)=\frac{v^2}{8}\cdot\frac{\left[2\left[1+T(1-x)-y\left(1+T-\theta_y\right)\right]-\theta_y\right]^2}{\left[1+T(1-x)-y\left(1+T-\theta_y\right)\right]^2}$$

$$u_g^{NTR}\left(q_g^{NTR^*};p^{NTR^*}\right)=\frac{v^2}{8}\cdot\frac{\left[1+T(1-2x)-2y\left(1+T-\theta_y\right)\right]^2}{\left[1+T(1-x)-y\left(1+T-\theta_y\right)\right]^2}$$

Hence, we can write:

$$\Delta SWF = x\left[\frac{v^2}{8}-\frac{v^2}{8}\cdot\frac{\left[1+2\left[T(1-x)-y\left(1+T-\theta_y\right)\right]\right]^2}{\left[1+T(1-x)-y\left(1+T-\theta_y\right)\right]^2}\right] + y\left[\frac{v^2}{8}-\frac{v^2}{8}\cdot\frac{\left[2\left[1+T(1-x)-y\left(1+T-\theta_y\right)\right]-\theta_y\right]^2}{\left[1+T(1-x)-y\left(1+T-\theta_y\right)\right]^2}\right] + z\left[\frac{v^2}{8}-\frac{v^2}{8}\cdot\frac{\left[1+T(1-2x)-2y\left(1+T-\theta_y\right)\right]^2}{\left[1+T(1-x)-y\left(1+T-\theta_y\right)\right]^2}\right] < 0$$

$$(x+y+z)\frac{v^2}{8} - \frac{v^2}{8}\left[x\cdot\frac{\left[1+2\left[T(1-x)-y\left(1+T-\theta_y\right)\right]\right]^2}{\left[1+T(1-x)-y\left(1+T-\theta_y\right)\right]^2}+y\cdot\frac{\left[2\left[1+T(1-x)-y\left(1+T-\theta_y\right)\right]-\theta_y\right]^2}{\left[1+T(1-x)-y\left(1+T-\theta_y\right)\right]^2}+z\cdot\frac{\left[1+T(1-2x)-2y\left(1+T-\theta_y\right)\right]^2}{\left[1+T(1-x)-y\left(1+T-\theta_y\right)\right]^2}\right] < 0$$

Since by hypothesis $x + y + z = 1$; then:

$$\frac{v^2}{8}\left[1 - \frac{x\left[1 + 2\left[T(1-x) - y\left(1 + T - \theta_y\right)\right]\right]^2 + y\left[2\left[1 + T(1-x) - y\left(1 + T - \theta_y\right)\right] - \theta_y\right]^2 +}{\left[1 + T(1-x) - y\left(1 + T - \theta_y\right)\right]^2} \frac{+z\left[1 + T(1-2x) - 2y\left(1 + T - \theta_y\right)\right]^2}{\left[1 + T(1-x) - y\left(1 + T - \theta_y\right)\right]^2}\right] < 0$$

$\frac{v^2}{8}$ is positive by hypothesis.

$$\left[1 + T(1-x) - y\left(1 + T - \theta_y\right)\right]^2 < x\left[1 + 2\left[T(1-x) - y\left(1 + T - \theta_y\right)\right]\right]^2 + y\left[2\left[1 + T(1-x) - y\left(1 + T - \theta_y\right)\right] - \theta_y\right]^2 + z\left[1 + T(1-2x) - 2y\left(1 + T - \theta_y\right)\right]^2$$

If we define $a = 1 + T(1-x) - y\left(1 + T - \theta_y\right)$ and $b = xT + y\left(1 + T - \theta_y\right)$, we can rewrite the previous inequality as:

$$a^2 < x[a + T - b]^2 + y\left[2a - \theta_y\right]^2 + z[a - b]^2$$

$$a^2 < x[a^2 + 2a(T - b) + (T - b)^2] + y\left[4a^2 - 4a\theta_y + \theta_y{}^2\right] + z[a^2 - 2ab + b^2]$$

$$a^2 < (x + y + z)a^2 + x[2a(T - b) + (T - b)^2] + y\left[3a^2 - 4a\theta_y + \theta_y{}^2\right] + z[b^2 - 2ab]$$

Since $x + y + z = 1$, then:

$$x[2a(T - b) + (T - b)^2] + y\left[3a^2 - 4a\theta_y + \theta_y{}^2\right] + z[b^2 - 2ab] > 0$$

$$2xaT - 2xab + xT^2 - 2xTb + xb^2 + 3ya^2 - 4ya\theta_y + y\theta_y{}^2 + zb^2 - 2zab > 0$$

Since $a = 1 + T - b$ and $z = 1 - x - y$, therefore:

$$2xT(1 + T - b) - 2xb(1 + T - b) + xT^2 - 2xTb + xb^2 + 3y(1 + T - b)^2 - 4y\theta_y(1 + T - b) + y\theta_y{}^2 + (1 - x - y)b^2 - 2b(1 - x - y)(1 + T - b) > 0$$

$$2xT + 2xT^2 - 2xTb - 2xb - 2xTb + 2xb^2 + xT^2 - 2xTb + xb^2 + 3y(1 + T)^2 - 6yb(1 + T) + 3yb^2 - 4y\theta_y(1 + T - b) + y\theta_y{}^2 + b^2 - xb^2 - yb^2 - 2b(1 + T) + 2b^2 + 2xb(1 + T) - 2xb^2 + 2yb(1 + T) - 2yb^2 > 0$$

$$2xT + 3xT^2 - 4xTb + 3y(1 + T)^2 - 4yb(1 + T) - 4y\theta_y(1 + T - b) + y\theta_y{}^2 + 3b^2 - 2b(1 + T) > 0$$

Substituting $b = xT + y\left(1 + T - \theta_y\right)$:

$$2xT + 3xT^2 - 4xT\left[xT + y\left(1 + T - \theta_y\right)\right] + 3y(1 + T)^2 - 4y(1 + T)\left[xT + y\left(1 + T - \theta_y\right)\right] - 4y\theta_y(1 + T) + 4y\theta_y\left[xT + y\left(1 + T - \theta_y\right)\right] + y\theta_y{}^2 + 3\left[xT + y\left(1 + T - \theta_y\right)\right]^2 - 2(1 + T)\left[xT + y\left(1 + T - \theta_y\right)\right] > 0$$

$$2xT + 3xT^2 - 4x^2T^2 - 4xyT\left(1 + T - \theta_y\right) + 3y(1 + T)^2 - 4xyT(1 + T) - 4y^2(1 + T)^2 + 4y^2\theta_y(1 + T) - 4y\theta_y(1 + T) + 4xyT\theta_y + 4y^2\theta_y(1 + T) - 4y^2{\theta_y}^2 + y{\theta_y}^2 + 3x^2T^2 + 6xyT\left(1 + T - \theta_y\right) + 3y^2\left(1 + T - \theta_y\right)^2 - 2xT(1 + T) - 2y(1 + T)^2 + 2y\theta_y(1 + T) > 0$$

$$xT^2 - x^2T^2 - 2xyT\left(1 + T - \theta_y\right) + y(1 + T)^2 - 4y^2(1 + T)^2 + 8y^2\theta_y(1 + T) - 2y\theta_y(1 + T) - 4y^2{\theta_y}^2 + y{\theta_y}^2 + 3y^2\left(1 + T - \theta_y\right)^2 > 0$$

$$xT^2 - x^2T^2 - 2xyT\left(1 + T - \theta_y\right) + y(1 + T)^2 - 4y^2\left(1 + T - \theta_y\right)^2 + 3y^2\left(1 + T - \theta_y\right)^2 - 2y\theta_y(1 + T) + y{\theta_y}^2 > 0$$

$$xT^2 - x^2T^2 - 2xyT\left(1 + T - \theta_y\right) + y\left(1 + T - \theta_y\right)^2 - y^2\left(1 + T - \theta_y\right)^2 > 0$$

$$xT^2 - x^2T^2 - 2xyT\left(1 + T - \theta_y\right) + y(1 - y)\left(1 + T - \theta_y\right)^2 > 0$$

$$x^2T^2 - xT\left[T - 2y\left(1 + T - \theta_y\right)\right] - y(1 - y)\left(1 + T - \theta_y\right)^2 < 0$$

$$\Delta = T^2\left[T - 2y\left(1 + T - \theta_y\right)\right]^2 + 4y(1 - y)T^2\left(1 + T - \theta_y\right)^2$$

$$\Delta = T^2\left\{\left[T - 2y\left(1 + T - \theta_y\right)\right]^2 + 4y(1 - y)\left(1 + T - \theta_y\right)^2\right\} > 0$$

$$x_{1,2} = \frac{T\left[T - 2y\left(1 + T - \theta_y\right)\right] \pm T\sqrt{\left[T - 2y\left(1 + T - \theta_y\right)\right]^2 + 4y(1 - y)\left(1 + T - \theta_y\right)^2}}{2T^2}$$

$$\frac{T - 2y\left(1 + T - \theta_y\right) - \sqrt{\left[T - 2y\left(1 + T - \theta_y\right)\right]^2 + 4y(1 - y)\left(1 + T - \theta_y\right)^2}}{2T} < x < \frac{T - 2y\left(1 + T - \theta_y\right) + \sqrt{\left[T - 2y\left(1 + T - \theta_y\right)\right]^2 + 4y(1 - y)\left(1 + T - \theta_y\right)^2}}{2T}$$

Note that $T - 2y\left(1 + T - \theta_y\right) - \sqrt{\left[T - 2y\left(1 + T - \theta_y\right)\right]^2 + 4y(1 - y)\left(1 + T - \theta_y\right)^2} < 0$:

$$\left[T - 2y\left(1 + T - \theta_y\right)\right]^2 < \left[T - 2y\left(1 + T - \theta_y\right)\right]^2 + 4y(1 - y)\left(1 + T - \theta_y\right)^2$$

$4y(1 - y)\left(1 + T - \theta_y\right)^2 > 0$ which is always satisfied.

Instead, $\frac{T - 2y\left(1 + T - \theta_y\right) + \sqrt{\left[T - 2y\left(1 + T - \theta_y\right)\right]^2 + 4y(1 - y)\left(1 + T - \theta_y\right)^2}}{2T} < 1$:

$$\left[T - 2y\left(1 + T - \theta_y\right)\right]^2 + 4y(1 - y)\left(1 + T - \theta_y\right)^2 < \left[T + 2y\left(1 + T - \theta_y\right)\right]^2$$

$$4y(1 - y)\left(1 + T - \theta_y\right)^2 < 4Ty\left(1 + T - \theta_y\right)$$

$$(1 - y)\left(1 + T - \theta_y\right) < T$$

$$y > 1 - \frac{T}{1 + T - \theta_y}$$

Since $1 - \frac{T}{1+T-\theta_y} < 0 \Rightarrow \theta_y > 1$ by hypothesis, the previous inequality is always satisfied.

Therefore, if $0 < x < \frac{T-2y(1+T-\theta_y)+\sqrt{[T-2y(1+T-\theta_y)]^2+4y(1-y)(1+T-\theta_y)^2}}{2T}$ then $\Delta SWF < 0$.

Vice versa, if $\frac{T-2y(1+T-\theta_y)+\sqrt{[T-2y(1+T-\theta_y)]^2+4y(1-y)(1+T-\theta_y)^2}}{2T} < x < 1$ then $\Delta SWF > 0$.

APPENDIX G

We have to prove the following inequalities:

$$u_{b,x}^{DM}\left(q_{b,x}^{DM^*};p_{b,x}^*\right)-u_{b,x}^{NTR}\left(q_{b,x}^{NTR^*};p^{NTR^*}\right)<0$$

$$u_g^{DM}\left(q_g^{DM^*};p_g^*\right)-u_{b,y}^{NTR}\left(q_{b,y}^{NTR^*};p^{NTR^*}\right)>0 \text{ if } \theta_y>1+\frac{T(1-x-y)}{1-y}$$

$$u_g^{DM}\left(q_g^{DM^*};p_g^*\right)-u_{b,y}^{NTR}\left(q_{b,y}^{NTR^*};p^{NTR^*}\right)<0 \text{ if } \theta_y<1+\frac{T(1-x-y)}{1-y}$$

$$u_g^{DM}\left(q_g^{DM^*};p_g^*\right)-u_g^{NTR}\left(q_g^{NTR^*};p^{NTR^*}\right)>0$$

Given that:

$$u_g^{DM}\left(q_g^{DM^*};p_g^*\right)=u_{b,x}^{DM}\left(q_{b,x}^{DM^*};p_{b,x}^*\right)=u_{b,y}^{DM}\left(q_{b,y}^{DM^*};p_{b,y}^*\right)=u_{b,z}^{DM}\left(q_{b,z}^{DM^*};p_{b,z}^*\right)=\frac{v^2}{8}$$

$$u_{b,x}^{NTR}\left(q_{b,x}^{NTR^*};p^{NTR^*}\right)=\frac{v^2}{8}\cdot\frac{\left[1+2\left[T(1-x)-y\left(1+T-\theta_y\right)\right]\right]^2}{\left[1+T(1-x)-y\left(1+T-\theta_y\right)\right]^2}$$

$$u_{b,y}^{NTR}\left(q_{b,y}^{NTR^*};p^{NTR^*}\right)=\frac{v^2}{8}\cdot\frac{\left[2\left[1+T(1-x)-y\left(1+T-\theta_y\right)\right]-\theta_y\right]^2}{\left[1+T(1-x)-y\left(1+T-\theta_y\right)\right]^2}$$

$$u_g^{NTR}\left(q_g^{NTR^*};p^{NTR^*}\right)=\frac{v^2}{8}\cdot\frac{\left[1+T(1-2x)-2y\left(1+T-\theta_y\right)\right]^2}{\left[1+T(1-x)-y\left(1+T-\theta_y\right)\right]^2}$$

- $u_{b,x}^{DM}\left(q_{b,x}^{DM^*};p_{b,x}^*\right)-u_{b,x}^{NTR}\left(q_{b,x}^{NTR^*};p^{NTR^*}\right)<0$

$$1+T(1-x)-y\left(1+T-\theta_y\right)<1+2\left[T(1-x)-y\left(1+T-\theta_y\right)\right]$$

$$T(1-x)-y\left(1+T-\theta_y\right)>0$$

$$T(1-x-y)+y\left(\theta_y-1\right)>0$$

Since by hypothesis, $1-x-y=z>0$; and $\theta_y>1$, the previous inequality is always satisfied.

- $u_g^{DM}\left(q_g^{DM^*};p_g^*\right)-u_{b,y}^{NTR}\left(q_{b,y}^{NTR^*};p^{NTR^*}\right)>0$ if $\theta_y>1+\frac{T(1-x-y)}{1-y}$

$$1+T(1-x)-y\left(1+T-\theta_y\right)>2\left[1+T(1-x)-y\left(1+T-\theta_y\right)\right]-\theta_y$$
$$T(1-x-y)+y\left(\theta_y-1\right)-\left(\theta_y-1\right)<0$$

$$T(1-x-y)-(1-y)\left(\theta_y-1\right)<0$$

$$\theta_y > 1 + \frac{T(1-x-y)}{1-y}$$

Likewise, if $\theta_y < 1 + \frac{T(1-x-y)}{1-y}$ then $u_g^{DM}\left(q_g^{DM^*}; p_g^*\right) - u_{b,y}^{NTR}\left(q_{b,y}^{NTR^*}; p^{NTR^*}\right) < 0$.

Note that $1 + \frac{T(1-x-y)}{1-y} < 1 + T \Rightarrow 1 - x - y < 1 - y$, since by hypothesis $-x < 0$.

- $u_g^{DM}\left(q_g^{DM^*}; p_g^*\right) - u_g^{NTR}\left(q_g^{NTR^*}; p^{NTR^*}\right) > 0$

$$1 + T(1-x) - y\left(1 + T - \theta_y\right) > 1 + T(1-2x) - 2y\left(1 + T - \theta_y\right)$$

$$y\left(1 + T - \theta_y\right) + xT > 0$$

Since by hypothesis, $\theta_y < 1 + T$, the previous inequality is always satisfied.